Cascaded Heading Estimation with Phase Coasting

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Abstract – In this paper, a cascaded heading estimation is proposed for low cost single-frequency GPS receivers and patch antennas. The method performs a kinematic calibration of double difference carrier phase measurements, and a subsequent phase coasting without the need of ambiguity resolution. The algorithm was successfully tested on a car, where it enabled a heading accuracy of 0.2 degrees, which corresponds to a relative positioning accuracy of 5 mm.

Keywords – Heading estimation, Phase Coasting, Phase Calibration.

I. Introduction

The GNSS carrier phases can be tracked with millimeter accuracy but are ambiguous. The Least-Squares Ambiguity Decorrelation Adjustment (LAMBDA) algorithm [1] of Teunissen is widely used to solve the integer least-squares problem. Teunissen has also developed a constrained LAMBDA method in [2], which takes statistical and deterministic a priori knowledge on the baseline length into account and, thereby, improves the reliability of ambiguity resolution. Jurkowski et al. [3] additionally included soft constraints on the baseline heading and pitch to further reduce the search space volume. Multi-frequency code carrier linear combinations are another means to improve the reliability of ambiguity resolution by increasing the wavelength [4]. For low cost receivers and antennas, phase jumps and loss of locks prevent any reliable ambiguity resolution, and require an alternative approach for carrier phase positioning.

II. Cascaded Heading Estimation

This section describes a cascaded heading estimation with kinematic phase calibration and subsequent phase coasting.

A. Measurement model

In this paper, the pseudorange and carrier phase measurements of receiver r and satellite k at epoch n are modeled as

$$\begin{aligned} \rho_{r,n}^k =& \|\vec{x}_{r,n} - \vec{x}_n^k\| + c\left(\delta\tau_{r,n} - \delta\tau_{r,n}^k\right) \\ &+ I_{r,n}^k + T_{r,n}^k + m_{\rho_{r,n}^k} + b_{\rho_{r,n}^k} + \varepsilon_{\rho_{r,n}^k} \\ \lambda\varphi_{r,n}^k =& \|\vec{x}_{r,n} - \vec{x}_n^k\| + c\left(\delta\tau_{r,n} - \delta\tau_{r,n}^k\right) \\ &- I_{r,n}^k + T_{r,n}^k + \lambda N_r^k + m_{\varphi_{r,n}^k} + b_{\varphi_{r,n}^k} + \varepsilon_{\varphi_{r,n}^k}, \end{aligned} \tag{1}$$

with the receiver position $\vec{x}_{r,n}$, the satellite position $\vec{x}_n^k$, the receiver clock offset $c\delta\tau_{r,n}$, the satellite clock offset $c\delta\tau_{r,n}^k$, the ionospheric delay $I_{r,n}^k$, the tropospheric delay $T_{r,n}^k$, the code and phase multipath errors $m_{\{\rho,\varphi\}_{r,n}^k}$, the carrier phase integer ambiguity N_r^k, the code and phase biases $b_{\{\rho,\varphi\}_{r,n}^k}$ and the measurement code and phase noises $\varepsilon_{\{\rho,\varphi\}_{r,n}^k}$.

B. Initial heading estimation with undifferenced pseudoranges

A rough estimate of the absolute receiver position and clock offset is given by the iterative least-squares solution of (1) using undifferenced pseudoranges. The position estimates of two subsequent epochs enable the computation of the receiver velocity and the direction of movement. For the latter one, the "baseline" vector $\hat{\vec{x}}_{r,n} - \hat{\vec{x}}_{r,n-1}$ is transformed from the Earth-Centered Earth Fixed (ECEF) coordinate frame into the local East-North-Up (ENU) frame by first computing the geodetic coordinates from $\hat{\vec{x}}_{r,n-1}$, and then performing a linear transformation of the baseline vector from ECEF into ENU coordinates. This results in the mapping

$$\{\hat{\vec{x}}_{r,n}, \hat{\vec{x}}_{r,n-1}\} \rightarrow \{\hat{E}_{r,n}, \hat{N}_{r,n}\}, \tag{2}$$

and enables the calculation of a rough heading estimate from the east $\hat{E}_{r,n}$ and north $\hat{N}_{r,n}$ position components, i.e.

$$\hat{\nu}_{2,r,n}^{(\hat{\mathrm{E}},\hat{\mathrm{N}})} = \mathrm{atan}(\hat{E}_{r,n}/\hat{N}_{r,n}), \tag{3}$$

which is counted clock-wise with a zero degree heading in Northern direction.

C. Kalman filtering of horizontal position estimates

The initial heading estimate of (3) is very noisy, and for velocities up to 200 km/h and a time interval of 0.2 s severely affected by code multipath. Therefore, a Kalman filter shall be applied to the initial East and North position estimates to reduce the noise and multipath. For the "measurements", we assume the linear model

$$\boldsymbol{z}_{r,n} = \begin{bmatrix} \hat{E}_{r,n} \\ \hat{N}_{r,n} \end{bmatrix} = \boldsymbol{H}_n \boldsymbol{x}_{r,n} + \boldsymbol{v}_{r,n}, \tag{4}$$

with the geometry matrix $\boldsymbol{H}_n = (\mathbf{1}^{2\times 2}, \mathbf{0}^{2\times 2})$, the state vector $\boldsymbol{x}_{r,n} = (E_{r,n}, N_{r,n}, \dot{E}_{r,n}, \dot{N}_{r,n})^T$, the combined measurement noise and multipath $\boldsymbol{v}_{r,n} \sim (\mathbf{0}, \boldsymbol{R}_n)$, and the measurement noise covariance matrix $\boldsymbol{R}_n$. The dynamics of the receiver are captured by a linear state space model, which is given by

$$\boldsymbol{x}_{r,n} = \boldsymbol{\Phi}_n \boldsymbol{x}_{r,n-1} + \boldsymbol{w}_{r,n}, \tag{5}$$

with the state transition matrix

$$\boldsymbol{\Phi}_n = \begin{bmatrix} 1 & dt \\ 0 & 1 \end{bmatrix} \otimes \mathbf{1}^{2\times 2}, \tag{6}$$

the time interval dt between subsequent epochs and the Gaussian distributed process noise $\boldsymbol{w}_{r,n} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{Q}_n)$ for covering the acceleration of the vehicle. Clearly, the process noise covariance matrix $\boldsymbol{Q}_n$ has to be chosen sufficiently large to be able to track the receiver dynamics. The a posteriori state estimate of the Kalman filter is given by

$$\begin{aligned}\hat{\boldsymbol{x}}_{r,n}^{+} &= \left[\hat{\tilde{E}}_{r,n},\ \hat{\tilde{N}}_{r,n},\ \hat{\dot{\tilde{E}}}_{r,n},\ \hat{\dot{\tilde{N}}}_{r,n}\right]^T \\ &= \hat{\boldsymbol{x}}_{r,n}^{-} + \boldsymbol{K}_n\left(\boldsymbol{z}_{r,n} - \boldsymbol{H}_n\hat{\boldsymbol{x}}_{r,n}^{-}\right),\end{aligned} \tag{7}$$

with the predicted state estimate $\hat{\boldsymbol{x}}_{r,n}^{-} = \boldsymbol{\Phi}_n\hat{\boldsymbol{x}}_{r,n-1}^{+}$, the Kalman gain $\boldsymbol{K}_n = \boldsymbol{P}_{\hat{\boldsymbol{x}}_n^-}\boldsymbol{H}_n^T(\boldsymbol{H}_n\boldsymbol{P}_{\hat{\boldsymbol{x}}_n^-}\boldsymbol{H}_n^T + \boldsymbol{R}_n)^{-1}$, and the covariance matrix $\boldsymbol{P}_{\hat{\boldsymbol{x}}_n^-}$ of the predicted state estimate. The smoothed heading is then obtained similar to (3) by

$$\hat{\nu}_{2,r,n}^{(\hat{\tilde{E}},\hat{\tilde{N}})} = \mathrm{atan}(\hat{\tilde{E}}_{r,n}/\hat{\tilde{N}}_{r,n}). \tag{8}$$

For low-cost receivers, some corrections are required to improve the integrity of the heading estimates:

First, the residuals $\boldsymbol{r}_{r,n} = \boldsymbol{z}_{r,n} - \boldsymbol{H}_n\hat{\boldsymbol{x}}_{r,n}^{-}$ are used to evaluate the a posteriori state estimate. If $\|\boldsymbol{r}_{r,n}\|^2$ exceeds a predefined threshold, the a posteriori state estimate is replaced by the state prediction.

Secondly, the heading estimates of the two receivers provide some redundancy, which can also be used to further enhance the integrity rather than to improve the accuracy. We select the heading estimate which is smoother over time, i.e.

$$\hat{\nu}_{2,n}^{(\hat{\tilde{E}},\hat{\tilde{N}})} = \hat{\nu}_{2,\tilde{r},n}^{(\hat{\tilde{E}},\hat{\tilde{N}})} \quad \text{with} \quad \tilde{r} = \arg\min_r \left|\hat{\nu}_{2,r,n}^{(\hat{\tilde{E}},\hat{\tilde{N}})} - \hat{\nu}_{2,n-1}^{(\hat{\tilde{E}},\hat{\tilde{N}})}\right|. \tag{9}$$

The third integrity enhancement refers to a failure in the first integrity enhancement, i.e. the erroneous replacement of the state update by the state prediction, which occurs if high receiver dynamics are misinterpreted as multipath: Therefore, the threshold on $\|\boldsymbol{r}_{r,n}\|^2$ is not kept constant but continuously increased during subsequent state replacements to prevent permanent rejections of the state update.

D. Heading estimation with Kalman filtering of smoothed horizontal position estimates

So far, we have performed a separate filtering of the east and north components. This enabled a substantial reduction of the noise and multipath and a linear processing without the need of any initial heading estimate. This is a clear advantage over a direct heading estimation, which is a non-linear problem that typically needs an accurate initial heading estimate for linearization.

However, the separate processing of the east and north components does not ensure a smooth heading. Therefore, a second Kalman filter shall now be used for smoothing the heading estimate $\hat{\nu}_{2,n}^{(\hat{\tilde{E}},\hat{\tilde{N}})}$. This second Kalman filter includes the heading and the rate of heading as state parameters, and scales the process noise covariance matrix according to the velocity estimates, i.e. the process noise covariance matrix is increased for low velocities due to eventually higher vehicle dynamics. The a posteriori state estimates of the second Kalman filter are denoted by $\hat{\nu}_{2,n}^{(\hat{\tilde{\tilde{E}}},\hat{\tilde{\tilde{N}}})}$ and $\hat{\nu}_{2,n}^{(\hat{\tilde{\tilde{E}}},\hat{\tilde{\tilde{N}}})}$.

E. Calibration of double difference carrier phases

The carrier tracking of low cost receivers is not sufficiently reliable to perform integer ambiguity resolution. Therefore, the double difference carrier phases are calibrated w.r.t. the smoothed code based heading $\hat{\nu}_{2,n}^{(\hat{\tilde{\tilde{E}}},\hat{\tilde{\tilde{N}}})}$. The calibrated double difference carrier phase measurements are modeled as

$$\begin{bmatrix}\lambda\varphi_{12,n}^{12}\\ \vdots \\ \lambda\varphi_{12,n}^{1K}\end{bmatrix} = \tilde{\boldsymbol{H}}_n \cdot l \cdot \begin{bmatrix}\cos(\nu_{1,n})\cos(\nu_{2,n})\\ \cos(\nu_{1,n})\sin(\nu_{2,n})\\ \sin(\nu_{1,n})\end{bmatrix} + \boldsymbol{\varepsilon}_{\varphi_{12,n}}, \tag{10}$$

with the baseline length l between both receivers, the pitch angle $\nu_{1,n}$, the heading $\nu_{2,n}$, and the geometry matrix

$$\tilde{\boldsymbol{H}}_n = \begin{bmatrix}(\vec{e}_n^{\,12})^T\\ \vdots \\ (\vec{e}_n^{\,1K})^T\end{bmatrix}\boldsymbol{R}_{\mathrm{L}}^{\mathrm{G}}, \tag{11}$$

with $\boldsymbol{R}_{\mathrm{L}}^{\mathrm{G}}$ being the transformation of the local East-North-Up (ENU) frame into the global ECEF frame, and $\vec{e}_n^{\,1k}$ being the single difference of two unit vectors pointing from satellites 1 and k to the receiver, coordinatized in the ECEF-frame. Assuming that an a priori knowledge l_{ap} on the baseline length is available and that the pitch angle is $\nu_{1,n} \approx 0°$, the double difference carrier phases are calibrated to

$$\lambda\boldsymbol{\varphi}_n = \begin{bmatrix}\lambda\varphi_{12,n}^{12}\\ \vdots \\ \lambda\varphi_{12,n}^{1K}\end{bmatrix} = \tilde{\boldsymbol{H}}_n \cdot l_{\mathrm{ap}} \cdot \begin{bmatrix}\cos(\hat{\nu}_{2,n}^{(\hat{\tilde{\tilde{E}}},\hat{\tilde{\tilde{N}}})})\\ \sin(\hat{\nu}_{2,n}^{(\hat{\tilde{\tilde{E}}},\hat{\tilde{\tilde{N}}})})\\ 0\end{bmatrix}. \tag{12}$$

F. Heading estimation based on phase coasting

Once the double difference carrier phases are calibrated, a precise heading can be obtained by phase coasting without the need of any further code measurements. As the estimation of $\nu_{2,n}$ is a non-linear problem, the trigonometric functions are linearized around the estimate $\hat{\nu}_{2,n-1}^{(\varphi)}$ of the previous epoch:

$$\begin{aligned}\cos(\nu_{2,n}) &\approx \cos(\hat{\nu}_{2,n-1}^{(\varphi)}) - \sin(\hat{\nu}_{2,n-1}^{(\varphi)}) \cdot (\nu_{2,n} - \hat{\nu}_{2,n-1}^{(\varphi)})\\ \sin(\nu_{2,n}) &\approx \sin(\hat{\nu}_{2,n-1}^{(\varphi)}) + \cos(\hat{\nu}_{2,n-1}^{(\varphi)}) \cdot (\nu_{2,n} - \hat{\nu}_{2,n-1}^{(\varphi)}),\end{aligned} \tag{13}$$

which enables us to rearrange the double difference phases as

$$\begin{aligned}\lambda\tilde{\boldsymbol{\varphi}}_n &= \lambda\boldsymbol{\varphi}_n - l_{\mathrm{ap}}\tilde{\boldsymbol{H}}_n\begin{bmatrix}\cos(\hat{\nu}_{2,n-1}^{(\varphi)}) + \sin(\hat{\nu}_{2,n-1}^{(\varphi)})\hat{\nu}_{2,n-1}^{(\varphi)}\\ \sin(\hat{\nu}_{2,n-1}^{(\varphi)}) - \cos(\hat{\nu}_{2,n-1}^{(\varphi)})\hat{\nu}_{2,n-1}^{(\varphi)}\\ 0\end{bmatrix}\\ &= \tilde{\boldsymbol{h}}_n \cdot \nu_{2,n} + \boldsymbol{\varepsilon}_{\varphi_{12,n}},\end{aligned} \tag{14}$$

with

$$\tilde{\boldsymbol{h}}_n = l_{\mathrm{ap}}\tilde{\boldsymbol{H}}_n\begin{bmatrix}-\sin(\hat{\nu}_{2,n-1}^{(\varphi)})\\ \cos(\hat{\nu}_{2,n-1}^{(\varphi)})\\ 0\end{bmatrix}. \tag{15}$$

The least-squares estimate of $\nu_{2,n}$ then easily follows as

$$\hat{\nu}_{2,n}^{(\varphi)} = \left(\tilde{\boldsymbol{h}}_n^T\tilde{\boldsymbol{h}}_n\right)^{-1}\tilde{\boldsymbol{h}}_n^T\lambda\tilde{\boldsymbol{\varphi}}_n. \tag{16}$$

This estimate is then used to make a new linearization of $\lambda\varphi_n$ around $\hat{\nu}_{2,n}^{(\varphi)}$, and then to re-compute $\lambda\tilde{\varphi}_n$ and $\hat{\nu}_{2,n}^{(\varphi)}$. Further iterations might be made but simulations have shown that two iterations are in general sufficient.

G. *Re-calibration of double difference phase measurements*

The phase coasting is always used for heading estimation if the velocity of the vehicle is low and/ or in urban environments due to eventually large code multipath. For increasing velocities, the phase tracking becomes less reliable while the filtered code based heading becomes more accurate. Therefore, the filtered code based heading is set to the phase based heading if a standing or slowly moving vehicle accelerates and exceeds a predefined minimum velocity threshold. If the velocity is above the threshold, the phase-based heading is continuously compared to the filtered code based one, and reset to the later one if the difference between both heading estimates exceeds a predefined threshold.

III. Performance analysis

The heading determination was tested in two environments.

A. *Urban environment*

Two GPS L1 patch antennas were mounted on the roof of a car with a distance of 1.71 m, and connected to 2 u-blox LEA 6T receivers. There was no external reference clock used nor any timing synchronization link between both receivers.

Fig. 1 shows the path of the vehicle, which follows a road aligned in North-South direction and is surrounded by some large buildings. The velocity of the vehicle was below 30 km/ h throughout this track.

Fig. 1. Path of vehicle in urban environment in sections of 60 s with starting points marked in blue (first: TOW = 117191.0 s) and ending points marked in red: a road aligned in north-south direction was driven multiple times with turns at both ends. The shown path is based on the code measurements, which are severely affected by multipath. The much more precise carrier phases were used to determine the heading, which is shown in Fig. 3.

Fig. 2 shows the calibrated double difference carrier phases for the vehicle track of Fig. 1. The rapid changes of the double difference carrier phases indicate the turns at the end of the road. The noise of the double difference carrier phases lies in the order of 1 cm.

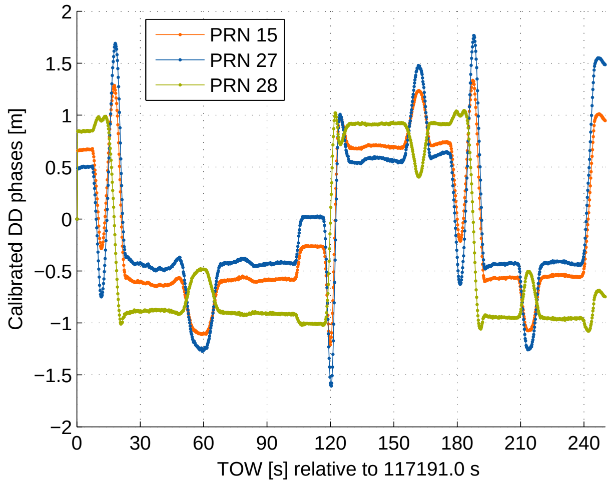

Fig. 2. Calibrated double difference carrier phase measurements correspond to the difference of two distances, which refer to the lengths of the baseline vector projections into the direction of two satellites. The noise of the double difference carrier phases lies in the order of 1 cm. Rapid changes of these double difference carrier phases indicate turns of the vehicle.

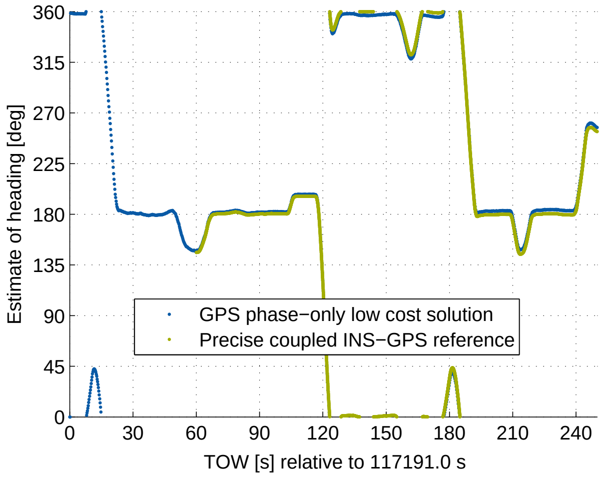

Fig. 3. Comparison of heading estimates of ANAVS's low cost cascaded system using phase coasting with an INS/GPS coupled reference system for the track of Fig 1: Both heading estimates match to a high degree.

Fig. 3 shows the obtained heading for the vehicle track of Fig. 1 using the calibrated double difference carrier phases of Fig. 2 and the proposed cascaded heading determination algorithm. One can observe that the heading estimate closely follows the estimate of a high precision INS/GPS-coupled reference system without a single outlier. It shall be noted that the reference system was never used for calibration of the low cost solution, which was performed solely once based on the filtered code based solution one minute before the reference system was switched on.

Fig. 4 shows the difference between the proposed low-cost heading determination system and a precise INS/GPS reference system over 1 minute: The difference is close to a white Gaussian noise process with a standard deviation of $0.18°$, which corresponds to a position uncertainty of 5 mm and reflects the irreducible carrier phase noise.

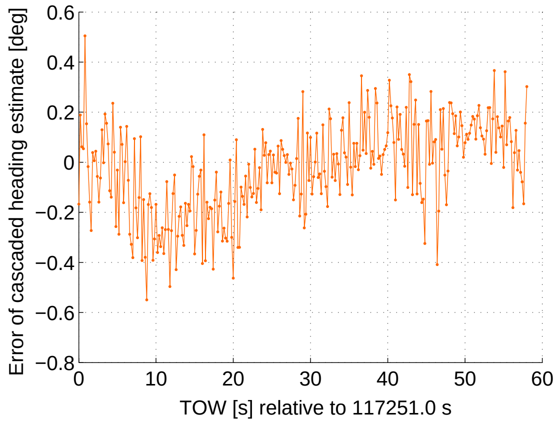

Fig. 4. Difference between heading estimates of ANAVS's low lost GPS phase-only solution and a precise INS/GPS reference system over 1 minute: The difference is close to a white Gaussian noise process with a standard deviation of $0.18°$, which corresponds to the double difference phase noise.

B. Rural environment

In a second test, the baseline between both receivers on the car was reduced to 1.08 m, and a path in a rural environment as shown in Fig. 5 was chosen. In the first 150 s, the vehicle was driving on a highway with a speed of around 110 km/h. The subsequent 120 s include a stop at the highway exit (180 s) and the driving through a roundabout (240 s).

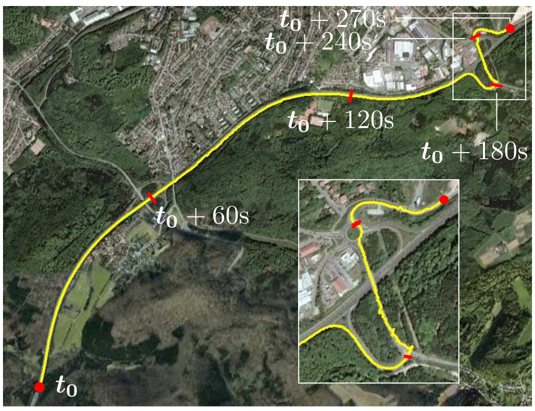

Fig. 5. Path of vehicle in rural environment with start at $\mathrm{TOW} = 120384.2$ s: In the first 150 s, the vehicle was driving on a highway with a speed of around 110 km/h. The subsequent 120 s include a stop at the highway exit (180 s) and the driving through a roundabout (240 s).

Fig. 6 shows the heading estimate of the proposed cascaded heading determination system for the vehicle track of Fig. 5. Frequent phase jumps were observed, but could be reliably detected and instantaneously corrected by extrapolating the respective double difference phases of the history. In the epochs marked by x in Fig. 6, the difference between the heading estimates of the phase coasting and the filtered code solution exceeded a threshold such that the double difference phases were re-calibrated to the filtered code based heading. The obtained heading is again closely following the INS/GPS reference solution throughout the test period. The high precision reference system was again only used for comparison but neither for any calibration nor correction of the proposed stand-alone low cost solution.

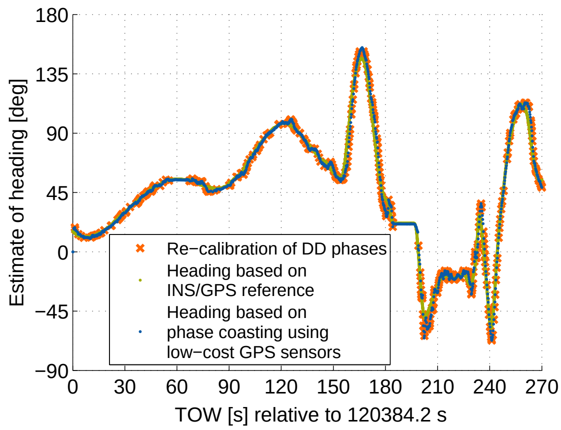

Fig. 6. Comparison of heading of proposed low cost cascaded system and of a high precision INS/GPS coupled reference system for a high vehicle speed: The heading of the low cost solution is again closely following the INS/GPS solution without a single outlier.

IV. Conclusion

In this paper, a cascaded heading estimation method was proposed for a pair of low cost GPS receivers and antennas. The heading is computed from a sequence of code based absolute position estimates and from coasting of double difference carrier phases. The method was successfully tested on a car, where the heading was determined with an accuracy of 0.2 deg, which corresponds to a position uncertainty of 5 mm.

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References

[1] P. Teunissen, *The least-squares ambiguity decorrelation adjustment: a method for fast GPS ambiguity estimation*, J. of Geodesy, vol. 70, pp. 65-82, 1995.
[2] P. Teunissen, *Integer least-squares theory for the GNSS compass*, J. of Geodesy, vol. 84, pp. 433-447, 2010.
[3] P. Jurkowski, P. Henkel, G. Gao, and C. Günther, *Integer Ambiguity Resolution with Tight and Soft Baseline Constraints for Freight Stabilization at Helicopters and Cranes*, Proc. of ION ITM, San Diego, USA, pp. 336-346, Jan. 2011.
[4] P. Henkel and C. Günther, *Reliable Integer Ambiguity Resolution: Multi-Frequency Code Carrier Linear Combinations and Statistical A Priori Knowledge of Attitude*, Navigation, Vol. 59, Issue 1, pp. 61-75, 2012.
[5] P. Henkel and P. Jurkowski, *Method for Determining the Heading of a Moving Object*, Europ. Patent Appl., EP 12 153 398.8, Jan. 2012.